A FUNCTIONAL-ANALYTIC METHOD FOR THE STUDY OF DIFFERENCE EQUATIONS

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We will give the generalization of a recently developed functional-analytic method for studying linear and nonlinear, ordinary and partial, difference equations in the ℓ_p^1 and ℓ_p^2 spaces, $p \in \mathbb{N}$, $p \geq 1$. The method will be illustrated by use of two examples concerning a nonlinear ordinary difference equation known as the Putnam equation, and a linear partial difference equation of three variables describing the discrete Newton law of cooling in three dimensions.

1. Introduction

The aim of this paper is to present the generalization of a functional-analytic method, which was recently developed for the study of linear and nonlinear difference equations of one, two, three, and four variables in the Hilbert space

$$\ell_p^2 = \left\{ f(i_1,\dots,i_p) : \mathbb{N}^p \longrightarrow \mathbb{C} : \sum_{i_1=1}^{\infty} \cdots \sum_{i_p=1}^{\infty} |f(i_1,\dots,i_p)|^2 < +\infty \right\} \tag{1.1}$$

and the Banach space

$$\ell_p^1 = \left\{ f(i_1,\dots,i_p) : \mathbb{N}^p \longrightarrow \mathbb{C} : \sum_{i_1=1}^{\infty} \cdots \sum_{i_p=1}^{\infty} |f(i_1,\dots,i_p)| < +\infty \right\}, \tag{1.2}$$

where $\mathbb{N}^p = \underbrace{\mathbb{N} \times \cdots \times \mathbb{N}}_{p\text{-times}}$ and $p = 1,2,3,4$.

More precisely, this method was introduced for the first time by Ifantis in [5] for the study of linear and nonlinear ordinary difference equations. Later, this method was extended by the authors in [7, 9, 10] in order to study a class of nonlinear ordinary difference equations more general than the one studied in [5]. For the study of linear and

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