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POLYNOMIAL SOLUTIONS OF LINEAR PARTIAL DIFFERENTIAL EQUATIONS

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Abstract. In this paper it is proved that the condition

$\lambda = a_1(n-2)(n-1)+\gamma_1(m-2)(m-1)+\beta_1(n-1)(m-1)+\delta_1(n-1)+\epsilon_1(m-1)$,

where $n=1,2,\ldots,N$, $m=1,2,\ldots,M$ is a necessary and sufficient condition for the linear partial differential equation

$$(a_1x^2+a_2x+a_3)u_{xx}+(\beta_1xy+\beta_2x+\beta_3y+\beta_4)u_{xy}$$
$$+(\gamma_1y^2+\gamma_2y+\gamma_3)u_{yy}+(\delta_1x+\delta_2)u_x+(\epsilon_1y+\epsilon_2)u_y=\lambda u,$$

where a_i, β_j, γ_i, δ_s, ϵ_s, $i=1,2,3$, $j=1,2,3,4$, $s=1,2$ are real or complex constants, to have polynomial solutions of the form

$$u(x,y)=\sum_{n=1}^{N}\sum_{m=1}^{M}u_{nm}x^{n-1}y^{m-1}.$$

The proof of this result is obtained using a functional analytic method which reduces the problem of polynomial solutions of such partial differential equations to an eigenvalue problem of a specific linear operator in an abstract Hilbert space. The main result of this paper generalizes previously obtained results by other researchers.