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Convexity results for the largest zero and functions involving the largest zero of q-associated polynomials

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We prove the convexity of the largest zero of the q-Lommel, the associated AL-Salam Carlitz II and the q-associated Laguerre polynomials as well as the convexity of products of certain functions with the largest zero of the q-associated Laguerre polynomials and associated Al-Salam–Carlitz II polynomials. Moreover, as a consequence of our results concerning the q-associated Laguerre polynomials, we find a recent result regarding the convexity of the function $(1/(\alpha+1))x_{n,1}(\alpha)$, where $x_{n,1}(\alpha)$ is the largest zero of the classical Laguerre $L_n^{\alpha}(x)$ polynomials. The method we use is a functional analytic one based on the three-term recurrence relations that the q-associated polynomials satisfy. By use of this method, the proofs of our results are straightforward.

Keywords: Convexity; q-Associated Laguerre polynomials; Associated Al-Salam–Carlitz II polynomials; q-Lommel polynomials

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