POLYNOMIAL SOLUTIONS OF LINEAR PARTIAL DIFFERENTIAL EQUATIONS

EUGENIA N. PETROPOULOU
Department of Engineering Sciences
Division of Applied Mathematics and Mechanics
University of Patras, 26500 Patras, Greece

PANAYIOTIS D. SIAFARIKAS
Department of Mathematics
University of Patras, 26500 Patras, Greece

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ABSTRACT. In this paper it is proved that the condition
\[ \lambda = a_1(n-2)(n-1) + \gamma_1(m-2)(m-1) + \beta_1(n-1)(m-1) + \delta_1(n-1) + \epsilon_1(m-1), \]
where \( n = 1, 2, ..., N \), \( m = 1, 2, ..., M \) is a necessary and sufficient condition for
the linear partial differential equation
\[
(a_1 x^2 + a_2 x + a_3) u_{xx} + (\beta_1 x y + \beta_2 x + \beta_3 y + \beta_4) u_{xy} \\
+ (\gamma_1 y^2 + \gamma_2 y + \gamma_3) u_{yy} + (\delta_1 x + \delta_2) u_x + (\epsilon_1 y + \epsilon_2) u_y = \lambda u,
\]
where \( a_i, \beta_j, \gamma_i, \delta_s, \epsilon_s, i = 1, 2, 3, j = 1, 2, 3, 4, s = 1, 2 \) are real or complex
constants, to have polynomial solutions of the form
\[ u(x, y) = \sum_{n=1}^{N} \sum_{m=1}^{M} u_{nm} x^{n-1} y^{m-1}. \]
The proof of this result is obtained using a functional analytic method which reduces
the problem of polynomial solutions of such partial differential equations
to an eigenvalue problem of a specific linear operator in an abstract Hilbert
space. The main result of this paper generalizes previously obtained results by
other researchers.