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HERMITIAN SYMMETRIC SPACES

AND

FREE FIELD TOPOLOGICAL THEORIES *

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ABSTRACT

We generalize the recent free field realization of $SU(N)$ current algebras by considering more general (super)groups G and their (super)Kählerian cosets G/H which are (super)Hermitian Symmetric Spaces. The method consists of introducing free fields corresponding to the (super)Kählerian coset and building the currents in a manner similar to Wigner's construction of the Poincaré group from generalized "orbital" and "spin" degrees of freedom. We then show that our parametrization of the currents yields free field representations for the superconformal $n = 2$ Kazama-Suzuki type models as well as the related topological quantum field theories.

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1-Introduction

During recent years there has been considerable effort in trying to understand the representation theory of current algebras by using free field realizations [1-12]. In this paper we generalize the coset method introduced in [1] for $SU(N{+}M)$, $SU(N,M)$, $SU(N/M)$ to all (super)groups G with Kählerian cosets G/H when G/H is a hermitian (super)symmetric space (HHS). In this case the product of subgroups $H=\prod_i H^i$ always contains only one factor of $U(1)$ while the rest of the subgroup is semi-simple. The complete list of hermitian symmetric spaces based on Lie groups appears in table-1, while the one for supergroups is in table-2. Although the tables list compact (super)groups our method applies to non-compact (super)groups as will be indicated below. Note that in the case of Lie groups, G_2, F_4, E_8 do not have the Kählerian structure we require.

The (super)Kählerian coset splits into two parts $(G/H)_+\oplus(G/H)_-$ which have opposite $U(1)$ charges that we have normalized to ±1. The generators associated with the two parts of the coset $(G/H)_\pm$ are hermitian conjugates of each other. Each part forms an irreducible representation of the subgroup as shown in table-1 by identifying the representation by its dimension. For example, for $SU(N+M)$ the coset $(G/H)_+=(N,\bar{M})_{+1}$ corresponds the cosets generators J_n^m, $n=1,2,\cdots,N$, $m=1,2,\cdots M$ which, under the subgroup $SU(N)\otimes SU(M)\otimes U(1)$, transforms like the fundamental representation of $SU(N)$, the complex conjugate fundamental representation of $SU(M)$ and carries charge $(+1)$ under $U(1)$. We have used a bar in $(\bar{M})$ to indicate the complex conjugate representation. The representations labelled as $N(N{\pm}1)/2$ are the symmetric $(+)$ and antisymmetric $(-)$ tensor representations of $SU(N)$, while $16,\bar{16}$ are the spinor representations of SO(10). The table also lists the Coxeter numbers g,h_i for the group G and subgroups H^i. The Coxeter number for the $U(1)$ factor is 0.

For the supergroups in table-2 the corresponding representations of $(G/H)_\pm$ and Coxeter numbers can also be listed[1]. In most of the paper we will concentrate on the case of Lie groups in order to keep the discussion as simple as possible. From ref. [1] where SU(N/M) is treated explicitly, it is seen that the results for supergroups closely parallel those of Lie groups by simply changing a few signs and using fermionic fields instead of bosonic fields in the appropriate expressions.

The Coxeter numbers are the same for compact or non-compact groups. For the non-compact version of the groups G with *compact* maximal subgroup $\prod_i H^i$, as listed in table-1, the unitary representation content of $(G/H)_\pm$ remains unchanged. We can also

[1] For example for $SU\big((n{+}p)/(m{+}q)\big)$, $g=n{+}p{-}m{-}q$; $h_i=(n{-}m),(p{-}q),0$; $sdim(G/H)=2(n-m)(p-q)$.

consider more general real forms for the non-compact groups G that allow the H^i to be non-compact as well. Then the structure of table-1 remains the same but $(G/H)_\pm$ will no longer correspond to unitary representations of H. In this case only minor modification in our treatment below is required (some signs or a metric as in [1]).

In section 2 we generalize the method of ref. [1] which is reminiscent of Wigner's construction of the 10 Poincaré generators in terms of orbital $(\vec{Q}, \vec{P})$, mass (M) and intrinsic spin $(\vec{S})$ operators. Our first step is to provide the analogs of Wigner's position, momentum, mass and spin variables for these algebras, and construct the Lie algebra generators from them. We assign the generalized "position-momentum" variables to the same representations as $(G/H)_+$ and $(G/H)_-$ respectively, as listed in table-1. The generalized "mass-spin" variables are in one to one correspondance to the generators of the subgroups H^i of table-1. We construct the generators of G from these more basic entities. We believe that, just as in the case of the Poincaré group, all representations of G can be described in this parametrization, but we will not attempt to give a proof of this fact which is related to induced representation theory.

All Casimir operators of G, and in particular the quadratic one, turn out to be independent from the "orbital position-momentum" operators and are constructed only from the generalized "mass-spin" subgroup variables (as for Poincaré group). When we consider the current algebra (i.e. affine G_k), this fact is intimately connected to the emergence of the "position-momentum" variables as free fields. In section 3 we construct the currents and the Sugawara stress tensor for G_k which is quadratic in the currents. It splits into several commuting parts associated with the subgroup factors $H^i_{k+g-h_i}$ plus a commuting coset part which is absent in the Lie algebra Casimir. The coset piece contains our "position-momentum" variables as generalized $\gamma - \beta$ free fields of the type introduced by Wakimoto [2]. This fact allows us to obtain free field representations of current algebra cosets of the type $G_k/\prod_i' H^i_{k+g-h_i}$ where the prime in the product indicates that the $U(1)$ factor is omitted in the denominator.

In section 4 we outline applications of our methods to free field representations of $n = 2$ superconformal Kazama-Suzuki models and the related topological quantum field theories. Finally we end the paper with concluding remarks and discussion in section 5.

2–Lie algebras

Consider a (super)group G with generators $J_A = (J_0, J_a, J_{+\alpha}, J_-^\alpha)$, where J_0 generates U(1), J_a, $a = 1, \cdots, (dimH - 1)$ generates the rest of the subgroup and $J_{+\alpha}, J_-^\alpha$, $\alpha =$

$1, \cdots, \frac{dim(G/H)}{2}$ correspond to the Kählerian coset as in table-1[2]. We will distinguish between upper and lower indices as they correspond to complex conjugate representations of the subgroup, as listed in table-1. We denote by $(t_a)_\alpha{}^\beta$ the matrix representation of the generator J_a when acting on the representation associated with $(G/H)_+$ [3].

The commutation relations are

$$
\begin{aligned}
&[J_a, J_b] = i f_{ab}{}^c J_c, \qquad [J_0, J_{+\alpha}] = +J_{+\alpha}, \qquad [J_0, J_-^\alpha] = -J_-^\alpha \\
&[J_a, J_{+\alpha}] = -(t_a)_\alpha{}^\beta J_{+\beta}, \qquad [J_a, J_-^\alpha] = (t_a)_\beta{}^\alpha J_-^\beta \\
&\epsilon[J_{+\alpha}, J_-^\beta] = -2(t_a)_\alpha{}^\beta J^a + \frac{2g\delta_\alpha{}^\beta}{dim(G/H)} J_0,
\end{aligned}
\tag{1}
$$

where $f_{ab}{}^c$ are the stucture constants of the (semi-)simple subgroup. Here $\epsilon = \pm$ depending on whether the group is compact $(+)$ or non-compact $(-)$ (when the real form is such that H=compact). The rest of the commutators are zero. The above commutation rules are invariant under hermitian conjugation. The normalization of the generators is such that, in a basis labelled by $J_A = (J_0, J_a, J_{+\alpha}, J_{-\bar{\alpha}})$, the non-zero entries in the Killing metric of the group G, i.e. $g_{AB} = f_{AC}{}^D f_{BD}{}^C$, takes the form

$$
g_{00} = dim(G/H), \qquad g_{ab} = g\delta_{ab}, \qquad g_{\alpha\bar{\beta}} = g_{\bar{\beta}\alpha} = 2\epsilon g \delta_{\alpha\bar{\beta}}, \tag{2}
$$

where g is the Coxeter number of the group G (see table-1) and the unit matrix $\delta_{\alpha\bar{\beta}}$ is used to take an unbarred upper greek index to a barred lower index, e.g. $J_{-\bar{\beta}} = J_-^\alpha \delta_{\alpha\bar{\beta}}$. We have chosen to write our commutation rules in eq.(1) in terms of unbarred indices only. This normalization requires that the matrix t_a and structure constants $f_{ab}{}^c$ satisfy

$$
Tr(t_a t_b) = \frac{1}{2}(g - h_i)\delta_{ab}, \qquad f_{ad}{}^c f_{bc}{}^d = -h_i \delta_{ab}. \tag{3}
$$

When the indices a, b, c, d belong to the subgroup H^i then h_i is the corresponding Coxeter number as listed in table-1. Furthermore, for the Kählerian cases that we consider one can derive the identity [12]

[2] In the case of supergroups of the table-2 some of the J_a and some of the $J_{+\alpha}, J_-^\alpha$ will be fermionic.

[3] For the compact groups of table-1, and their non-compact counterparts with the same maximal *compact* subgroups, the matrix t_a can be chosen to be hermitian $t_a^\dagger = t_a$. For a more general real form with some of the subgroups H^i non-compact this matrix is pseudo-hermitian (i.e. hermitian up to a conjugation with the metric of the non-compact group). Since this is a minor modification we will concentrate mainly on the compact and non-compact G that have the H^i compact, so that t_a is hermitian.

$$\frac{g}{2}dim(G/H) = \sum_i (g - h_i) dim(H^i). \tag{4}$$

This useful identity that we will use later can be explicitly verified directly from table-1. It can be shown that the commutation rules given above obey the Jacobi identities provided

$$(t_d)_\alpha{}^\gamma (t_d)_\beta{}^\epsilon - (t_d)_\beta{}^\gamma (t_d)_\alpha{}^\epsilon = \frac{g}{dim(G/H)}(\delta_\alpha{}^\epsilon \delta_\beta{}^\gamma - \delta_\alpha{}^\gamma \delta_\beta{}^\epsilon), \tag{5}$$

which is consistent with the normalizations used above.

The quadratic Casimir follows from the form of the inverse Killing metric

$$\begin{aligned} C_2^G &= J_a J_a + \frac{\epsilon}{2}(J_-^\alpha J_{+\alpha} + J_{+\alpha} J_-^\alpha) + \frac{g}{dim(G/H)} J_0^2 \\ &= J_a J_a + \epsilon J_{+\alpha} J_-^\alpha + \frac{g}{dim(G/H)} J_0^2 - \frac{g}{2} J_0, \end{aligned} \tag{6}$$

which we have rearranged by using the identity $J_{+\alpha} J_-^\alpha = J_-^\alpha J_{+\alpha} + \epsilon g J_0$.

We now follow [1] to extend the work of [13] on induced representations and then consider the infinitesimal form which allows us to express the generators of G in terms of generalized "orbital" and "spin" operators. The basic operators are the (complex) generalized "position-momentum" operators Q_α, P^α which are classified in the same representations as $(G/H)_\pm$ and the "spin" operators S_a, S_0 that stand in one-to-one correspondance to the generators of the subgroups $\prod_i H^i$. The defining commutation relations for these are

$$[P^\alpha, Q_\beta] = \delta^\alpha_\beta, \qquad [S_a, S_b] = i f_{ab}{}^c S_c, \tag{7}$$

while all other commutators vanish. In particular note that S_0 commutes with all operators, just like the mass operator in the Poincaré case. In terms of this set we find the following structure for the generators of G

$$\begin{aligned} &J_-^\alpha = \epsilon P^\alpha, \qquad J_0 = S_0 + Q_\alpha P^\alpha, \qquad J_a = S_a + (-t_a)_\beta{}^\delta Q_\delta P^\beta \\ &J_{+\alpha} = -\frac{2g}{dim(G/H)} Q_\alpha (S_0 + \frac{1}{2} Q_\beta P^\beta) + 2(t_a)_\alpha{}^\gamma Q_\gamma (S_a + \frac{1}{2}(-t_a)_\beta{}^\delta Q_\delta P^\beta). \end{aligned} \tag{8}$$

With the help of (5),(7) we can easily prove that this construction of the generators of G indeed obey the Lie algebra of eq.(1).

We can now compute the quadratic Casimir operator in terms of our basic operators $(S_0, S_a, Q_\alpha, P^\alpha)$ by substituting (8) into (6)

$$C_2^G = S_a S_a + \frac{g}{dim(G/H)} S_0^2 - \frac{g}{2} S_0. \tag{9}$$

Remarkably, we find that all dependence on the "orbital" operators Q_α, P^α drops out and that C_2^G is entirely determined by the "spin" operators (as in the Poincaré case). This is a general result for all higher Casimir operators $C_p,\ p = 2, 3, \cdots$ as well, and it can be argued as follows: In C_p we must have P^α always accompanied with Q_α in order to insure that C_p is neutral under the $U(1)$ transformations, i.e. $[C_p, J_0] = 0$. In addition, since any Casimir operator C_p must commute with any generator, it must in particular commute with P^α which is one of the generators according to eq.(8). This requires $[C_p, P^\alpha] = 0$ which is possible only if C_p is independent of Q_α. Therefore, C_p cannot depend on the "orbital" operators Q_α, P^α. The expressions for higher Casimirs in terms of the spin operators will be given elsewhere.

We shall see in the next section that the stress tensor in the case of the current algebra retains the form of the Casimir operator in (9) with the important addition that the "orbital" part doesn't completely drop out due to normal ordering, but instead it contributes an additional term proportional to the stress tensor of the free fields $(\beta^\alpha, \gamma_\alpha) \sim (P^\alpha, Q_\alpha)$ or "ghost" system. Therefore the origin for the emergence of free fields can be traced to the absence of the orbital operators in the expression of the Casimirs.

3- Current algebras

We consider current algebras in 1+1 dimensions corresponding to WZNW models at integer level k for the kind of groups of the previous section. The holomorphic currents $J_A(z) = (J_0(z),\ J_a(z),\ J_{+\alpha}(z),\ J_-^\alpha(z))$ are lebelled as their Lie algebra counterparts. Their operator product expansion (OPE) is

$$\begin{aligned}
&J_a(z)J_b(w) = \frac{i f_{ab}{}^c J_c(w)}{z-w} + \frac{\frac{k}{2}\delta_{ab}}{(z-w)^2} \\
&J_0(z)J_{+\alpha}(w) = \frac{+J_{+\alpha}(w)}{z-w}, \qquad J_0(z)J_-^\alpha(w) = \frac{-J_-^\alpha(w)}{z-w} \\
&J_a(z)J_{+\alpha}(w) = -\frac{(t_a)_\alpha{}^\beta J_{+\beta}(w)}{z-w}, \qquad J_a(z)J_-^\alpha(w) = \frac{(t_a)_\beta{}^\alpha J_-^\beta(w)}{z-w} \\
&\epsilon J_{+\alpha}(z)J_-^\beta(w) = \frac{1}{z-w}\left[-2(t_a)_\alpha{}^\beta J_a(w) + \frac{2g}{dim(G/H)}\delta_\alpha{}^\beta J_0(w)\right] + \frac{k\delta_\alpha{}^\beta}{(z-w)^2} \\
&J_0(z)J_0(w) = \frac{k\ dim(G/H)}{2g}\frac{1}{(z-w)^2} \qquad .
\end{aligned} \tag{10}$$

To be consistent with the standard notation in the literature we will rename $Q \to \gamma, P \to \beta$. They have conformal weights 0 and 1 respectively. In terms of our $S_0, S_a, \gamma_\alpha, \beta^\alpha$ basis the currents can be written by using (8) as

$$\begin{aligned}
&J_-^\alpha(z) = \epsilon\beta^\alpha(z)\\
&J_0(z) = S_0(z) + :\gamma_\alpha(z)\beta^\alpha(z):\\
&J_a(z) = S_a(z) + (-t_a)_\alpha{}^\beta :\gamma_\beta(z)\beta^\alpha(z):\\
&J_{+\alpha} = -\frac{2g}{dim(G/H)} :\gamma_\alpha(z)(S_0(z) + \frac{1}{2}\gamma_\delta(z)\beta^\delta(z)):\\
&+ 2(t_a)_\alpha{}^\lambda :\gamma_\lambda(z)(S_a(z) + \frac{1}{2}(-t_a)_\delta{}^\beta\gamma_\beta(z)\beta^\delta(z)): + k\partial\gamma_\alpha(z),
\end{aligned} \tag{11}$$

where the last term in $J_{+\alpha}$ is needed in order to produce the correct level in (10), and it may be interpreted as arising from the normal ordering of the cubic term.
The conjugate fields $\gamma_\alpha, \beta^\alpha$ obey the following OPE

$$\beta^\alpha(z)\gamma_\lambda(w) = \frac{\delta_\lambda{}^\alpha}{z-w} \quad . \tag{12}$$

Consistency of (10-12) requires that the spin currents obey the OPE

$$\begin{aligned}
S_0(z)S_0(w) &= \frac{1}{2g}dim(G/H)(k+g)\frac{1}{(z-w)^2}\\
S_a(z)S_b(w) &= \frac{if_{ab}{}^c S_c(w)}{z-w} + \frac{\frac{1}{2}\delta_{ab}(k+g-h_i)}{(z-w)^2} \quad ,
\end{aligned} \tag{13}$$

with all the remaining OPE between the basic fields $(S_0, S_a, \beta^\alpha, \gamma_\alpha)$ zero. Note the level of the $S_a(z)$ current is $k+g-h_i$ for each subgroup H^i and that of S_0 is $\frac{(k+g)dim(G/H)}{g}$.

We may now calculate the stress energy momentum tensor $T_k^G = \frac{{}^\circ_\circ J_A J^A {}^\circ_\circ}{k+g}$ which is analogous to the quadratic Casimir operator of eq.(6). The circles indicate normal ordering of non-abelian currents. In this computation we must take into account the singular nature of the OPE and extract the finite part of the OPE $J_A(z)J^A(w)$. The result is paralel to eq.(9) except for the derivative terms below

$$\begin{aligned}
&T_k^G = {\sum_i}' T^{H^i}_{k+g-h_i} + T_{S_0} + T_{\beta\gamma}\\
&{\sum_i}' T^{H^i}_{k+g-h_i} = \frac{{}^\circ_\circ S_a S_a {}^\circ_\circ}{k+g}, \qquad T_{S_0} = \frac{g\,{}^\circ_\circ S_0 S_0 {}^\circ_\circ}{dim(G/H)(k+g)} + \frac{g}{2(k+g)}\partial S_0\\
&T_{\beta\gamma} =: \beta^\alpha\partial\gamma_\alpha :,
\end{aligned} \tag{14}$$

where the prime in the sum means that we have excluded from the summation the U(1) piece. The stress tensor of the U(1) part may be rewritten in terms of a free scalar field ϕ with a background charge

$$T_{S_0} = \frac{1}{2} : (\partial\phi)^2 : + \alpha_0 \partial^2 \phi, \qquad S_0 = \sqrt{\frac{dim(G/H)}{2g}(k+g)}\; \partial\phi, \tag{15}$$

where the background charge is defined as $\alpha_0 = \frac{1}{2}\sqrt{\frac{g\ dim(G/H)}{2(k+g)}}$. The central charges of the three commuting parts of the energy momentum tensor are

$$c_{S_a} = \frac{\sum_i'(k+g-h_i)dimH^i}{k+g}, \quad c_{S_0} = 1 - \frac{3dim(G/H)g}{2(k+g)}, \quad c_{\beta\gamma} = dim(G/H). \tag{16}$$

As a consistency check the sum of the above central charges should add up to the central charge of the group G, $c_G = \frac{k\ dimG}{k+g}$. Indeed $c_{S_a} + c_{S_0} + c_{\beta\gamma} = c_G$ after we make use of (4). Also we can easily check using (14),(15) that the currents, and in particular $J_{+\alpha}$, as parametrized in (11) tansform as conformal tensors of weight one.

We have thus managed to express the stress tensor in terms of the free fields $\beta,\ \gamma,\ \phi$ and the non-abelian spin current $S_a(z)$ of eq.(13). If one wishes, it is possible to aplly the same method to the currents $S_a(z)$ and reduce the entire system to free fields. However, as seen in the next section this further step is not useful in certain applications.

4-Relation to Kazama-Suzuki and topological models

There is similarity in the structure of our coset models and the Kazama-Suzuki $n = 2$ superconformal models [14] based on a semi-simple group G at level k when the latter are written in the following form [12]

$$\frac{G_k \times \prod_i H^i_{g-h_i}}{\prod_i H^i_{k+g-h_i}}, \tag{17}$$

where the currents in $\prod_i H^i_{g-h_i}$ in the numerator are constructed from free fermions, of conformal dimension one half, that have coset $(G/H)_\pm$ quantum numbers, i.e. $\psi_\alpha, \bar\psi^\alpha$, and are in one to one correspondance to our $\gamma_\alpha, \beta^\alpha$. Their OPE is $\psi_\alpha(z)\bar\psi^\beta(w) = \frac{\delta_\alpha{}^\beta}{z-w}$. The stress tensor for the fermions may be constructed from the fermionic currents

$$J^\psi_a = (-t_a)_\alpha{}^\beta : \psi_\beta \bar\psi^\alpha :, \qquad J^\psi_0 =: \psi_\alpha \bar\psi^\alpha :, \tag{18}$$

which have levels $g - h_i$ and $dim(G/H)$ respectively. Namely

$$
\begin{aligned}
T_{\psi\bar{\psi}} &= \frac{{}^\circ_\circ J^\psi_a J^\psi_a {}^\circ_\circ}{g} + \frac{{}^\circ_\circ J^\psi_0 J^\psi_0 {}^\circ_\circ}{dim(G/H)} \\
&= \frac{1}{2}(:\partial\bar{\psi}^\alpha \psi_\alpha : - : \bar{\psi}^\alpha \partial\psi_\alpha :),
\end{aligned}
\tag{19}
$$

where the second line has been proven in [12], and is in aggrement with what is expected for the stress tensor of two dimension one half conjugate fields [19].

The currents in $\prod_i H^i_{k+g-h_i}$ in the denominator are the subgroup currents given in the third line of (11) plus the corresponding fermionic terms. Namely $\tilde{J}_a = J_a + J^\psi_a$, $\tilde{J}_0 = J_0 + J^\psi_0$. The level of the fermionic terms cancels the level of the bosonic (β, γ) terms so that the total level is that of the spin currents S_a, S_0 namely $k + g - h_i$, $\frac{dim(G/H)}{g}(k+g)$ respectively. The stress tensor for the currents in the denominator is then

$$
\tilde{T} = \frac{{}^\circ_\circ \tilde{J}_a \tilde{J}_a {}^\circ_\circ}{k+g} + \frac{g}{dim(G/H)} \frac{{}^\circ_\circ \tilde{J}_0 \tilde{J}_0 {}^\circ_\circ}{k+g} \quad . \tag{20}
$$

The stress tensor of the Kazama-Suzuki model (17) is therefore

$$
\begin{aligned}
T &= T^G_k + T_{\psi\bar{\psi}} - \tilde{T} \\
&= T_{\beta\gamma} + T_{\psi\bar{\psi}} + \frac{{}^\circ_\circ S_a S_a {}^\circ_\circ - {}^\circ_\circ \tilde{J}_a \tilde{J}_a {}^\circ_\circ}{k+g} + \frac{g(: S_0 S_0 : - {}^\circ_\circ \tilde{J}_0 \tilde{J}_0 {}^\circ_\circ)}{dim(G/H)(k+g)} + \frac{g}{2(k+g)} \partial S_0,
\end{aligned}
\tag{21}
$$

where we used the decomposition given in eq.(14). Suppose now that we consider a constrained sector of free-field Fock space in which the total currents $\tilde{J}_a, \tilde{J}_0$ are required to be weakly equal to the spin currents S_a, S_0 (except for zero modes). The appropriate weak constraints to be applied on kets are then

$$
(t_a)_\alpha{}^\beta (: \gamma_\beta \beta^\alpha : + : \psi_\beta \bar{\psi}^\alpha :) \simeq 0, \qquad : \gamma_\alpha \beta^\alpha : + : \psi_\alpha \bar{\psi}^\alpha : \simeq 0. \tag{22}
$$

These maybe interpreted as zero level gauge currents corresponding to a gauge invariance $\prod_i H^i_0$. The constraints of eq.(22) are then simply the requirement for gauge invariant states[4]. This is consistent with the idea that zero level currents are zero in the appropriate Hilbert space. In this gauge invariant sector the energy momentum tensor simplifies since, except for zero modes $S_a \simeq \tilde{J}_a \simeq 0$, $S_0 \simeq \tilde{J}_0 \simeq 0$. Thus, when eq.(22) holds he Kazama-Suzuki models based on hermitian symmetric spaces are reduced to a free theory, whose stress tensor consists of three commuting parts, namely

[4] We call these constraints weak since their OPE are nonzero, and only the positive modes $(n \geq 1)$ annihilate kets. We will use for any weak equality of this type the symbol $\simeq$.

$$T \simeq: \beta^\alpha \partial\gamma_\alpha : + \frac{1}{2}(: \partial\bar{\psi}^\alpha \psi_\alpha : - : \bar{\psi}^\alpha \partial\psi_\alpha :) + \alpha_0 \partial^2 \phi. \tag{23}$$

Their contribution to the central charge is $dim(G/H)$, $\frac{1}{2}dim(G/H)$, $-\frac{3g\ dim(G/H)}{2(k+g)}$ respectively. The sum can be easily seen to equal $c = \frac{3k\ dim(G/H)}{2(k+g)}$ which is the expected expression. The zero modes of $S_a, S_0, \gamma_0, \beta_0, \psi_0, \bar{\psi}_0$ play an important role in determining the dimension operator L_0 and the grade 0 states, as will be explained in detail elsewhere.

Furthermore our approach seems to be well suited for investigating topological field theories obtained by twisting $n = 2$ superconformal theories [15-17]. To obtain these models one defines a new stess tensor $\hat{T}$ as follows

$$\hat{T} = T + \frac{1}{2}\partial J, \tag{24}$$

where $J(z)$ is the $U(1)$ current of the $n = 2$ superalgebra which, for HSS spaces, is realized as follows [12,14]

$$J = \frac{1}{k+g}(kJ_0^\psi - gJ_0). \tag{25}$$

One may verify that $\hat{T}$ generates a Virassoro algebra with vanishing central charge $\hat{c} = 0$. As a result of this modification the supercurrents $G_\pm(z)$ aquire dimensions 1 and 2 respectively. Also the $U(1)$ current $J(z)$ is no longer conformal with respect to $\hat{T}$. Let us now find how the expressions for the various $n = 2$ supercurrents are simplified on the space where (22) holds. Using the expressions for $G_\pm(z)$ from [12,14] and (25),(24),(22),(23),(5),(11) we find

$$\begin{aligned}
&J \simeq J_0^\psi - \frac{g}{k+g}S_0 \\
&G_+ = \frac{\epsilon}{\sqrt{k+g}}J_-^\alpha \psi_\alpha = \frac{1}{\sqrt{k+g}}\beta^\alpha\psi_\alpha \\
&G_- = \frac{1}{\sqrt{k+g}}J_{+\alpha}\bar{\psi}^\alpha \simeq \frac{1}{\sqrt{k+g}}\big(-\frac{2g}{dim(g/H)}\gamma_\delta\bar{\psi}^\delta S_0 + 2\bar{\psi}^\alpha (t_a)_\alpha{}^\delta \gamma_\delta S_a + k\partial\gamma_\alpha\bar{\psi}^\alpha\big) \\
&\hat{T} \simeq \lambda : \beta^\alpha\partial\gamma_\alpha : -(1-\lambda) : \partial\beta^\alpha\gamma_\alpha : -\lambda : \bar{\psi}^\alpha\partial\psi_\alpha : +(1-\lambda) : \partial\bar{\psi}^\alpha\psi_\alpha : .
\end{aligned} \tag{26}$$

It is remarkable that the ϕ dependance of $\hat{T}$ dropped out completely!! With respect to the new stress tensor β and $\bar{\psi}$ have conformal dimensions [5] $\lambda = \frac{2k+g}{2(k+g)}$ whereas their conjugate

[5] For compact algebras $k \geq 0$ and $\frac{1}{2} \leq \lambda \leq 1$ whereas for the noncompact ones $k < -g$ [12] and $\lambda > 1$.

partners γ and ψ have conformal dimensions $1-\lambda = \frac{g}{2(k+g)}$, leading to the observation that we have arrived at a super $n=2$ ghost system of the type described in ref. [19][6]. Because $G_+(z)$ has new dimension one, it can be identified with the BRST generator $Q(z)$ [15,18] of the topological models. Then it can be easily seen that, the bosons and fermions form super multiplets under the BRST transformation $\delta A(z) = \oint dw G_+(w) A(z)$, where $A(z)$ is some field. Namely

$$\begin{aligned} \delta\gamma_\alpha &\sim \psi_\alpha, \qquad \delta\psi_\alpha = 0 \\ \delta\bar{\psi}^\alpha &\sim \beta^\alpha, \qquad \delta\beta^\alpha = 0. \end{aligned} \tag{27}$$

Such supersymmetry is reflected of course in the form of $\hat{T}$. It should be emphasized that the dependence of the superconformal algebra generators $J(z),\ G_-(z)$ on the spin currents $S_a(z),\ S_0(z)$ is only through the zero modes $S_{a,0},\ S_{0,0}$, namely the generators of the Lie algebra. This is so because we work on the coset space eq.(17).

5-Discussion and concluding remarks

In this paper we managed to express the current algebras on the Kählerian cosets $G_k/(\prod_i H^i_{k+g-h_i})$ in terms of free fields. We have treated compact and non-compact current algebras on an equal footing. However their Hilbert spaces are quite different as in the case of the Lie algebra.

It is possible to extend quite easily our construction to cover the supergroups of table-2 with super-Kählerian cosets. In this case some of the generators are of fermionic nature and subsequently the commutator of two such operators should be replaced with an anticommutator. It turns out that the corresponding β,γ fields are also fermionic in nature. For the coset $SU(N/M)/(SU(N)\times SU(M)\times U(1))$ this was done explicitly in [1]. It is evident that our methods apply in a straightforward way to all compact or noncompact groups and supergroups of tables 1,2.

For the compact current algebras of table-1 one knows that unitary Hilbert spaces exist and examples have been constructed in closely related free field approaches. However none of the previous work [2-10] have labeled their Hilbert space with our kind of "intrinsic spin" subgroups at the indicated level, except in Wakimoto's case for SU(2). Rewriting of the previous work should elucidate the structure of free field representations.

Non-compact current algebras probably do not have unitary representations but their cosets do [13,14]. It would be interesting and useful for model building to reconstruct these

[6] We differ from [19] by a minus sign in the $\hat{T}_{\beta\gamma}$ term because we have defined a $\beta-\gamma$ OPE which differs by a minus sign from that in [19].

in terms of our variables.

Beyond the known unitary Hilbert spaces indicated above, there is the possibility that our G/H theories may be unitary by themshelves for a certain range of k, $dimG$, $dimH$. Of course their unitary Hilbert spaces will be a subspace of the Fock space of the free fields.

We have seen that if we impose the constraints of eq.(22) namely, restrict ourselves to the singlet sector of the gauge currents $\prod_i H_0^i$, we can express $n = 2$ Kazama-Suzuki superconformal models and moreover topological models in terms of free fields in a natural way. There appears to be a supergeometry lying at the root of these models. We expect to be able to make new progress in this class of models by using our formalism.

With our method one can express higher order casimirs of Lie algebras and the closely related W-algebras in terms of the "orbital" and "intrinsic spin" variables. The "orbital" part will drop out as in the Lie algebra case but it will persist in the case of the W-algebra as it did in the case of the quadratic stress tensor. Work along the lines indicated above will be reported elsewhere.

G	$H = \prod_i H_i$	$(G/H)_+ \oplus (G/H)_-$	$dim(G/H)$	g	h_i
$SU(N+M)$	$S(U(N) \otimes U(M))$	$(N, \bar{M})_{+1} \oplus (\bar{N}, M)_{-1}$	$2NM$	$N+M$	$N, M, 0$
$SO(N+2)$	$SO(N) \otimes U(1)$	$(N)_{+1} \oplus (N)_{-1}$	$2N$	N	$N-2, 0$
$SO(2N)$	$SU(N) \otimes U(1)$	$(\frac{N(N-1)}{2})_{+1} \oplus (\frac{\overline{N(N-1)}}{2})_{-1}$	$N(N-1)$	$2N-2$	$N, 0$
$USp(2N)$	$SU(N) \otimes U(1)$	$(\frac{N(N+1)}{2})_{+1} \oplus (\frac{\overline{N(N+1)}}{2})_{-1}$	$N(N+1)$	$2N+2$	$N, 0$
E_6	$SO(10) \otimes U(1)$	$(16)_{+1} \oplus (\bar{16})_{-1}$	32	12	$8, 0$
E_7	$E_6 \otimes U(1)$	$(27)_{+1} \oplus (\bar{27})_{-1}$	54	18	$12, 0$

Table-1. Hermitian symmetric spaces for compact groups. G may be replaced by a non-compact group as described in the text.

G	$H = \prod_i H_i$
$SU(n+p/m+q)$	$SU(n/m) \otimes SU(p/q) \otimes U(1)$
$OSp(n+2/2m)$	$OSp(n/2m) \otimes U(1)$
$OSp(2n/2m)$	$SU(n/m) \otimes U(1)$
$P(2n-1)$	$SU(n/n) \otimes U(1)$
$Q(2n-1)$	$Q(n-1) \otimes Q(n-1) \otimes F \otimes U(1)$
$D(2,1;t)$	$SU(1/2) \otimes U(1)$
$F(4)$	$OSp(2/4) \otimes U(1)$

Table-2. Supergroups with super symmetric spaces.

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